Malaysia International Mathematics Olympiad Competition 2018

24 November, 2018
9.30 am – 11.00 am

- Upper Primary Paper -

Jointly Organised by :

Persatuan Matematik Olimpiad Malaysia (PERMATO)
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E Mathematics Olympiad System

马来西亚数学奥林匹克学会
力行国民型华文学校
黎明华文小学
数学奥林匹克学研中心
联合主办

Instructions:

- **Do not turn to the first page until you are told to do so.**
- **Write down your name, your contestant number and your school's name on the answer sheet.**
- **Write down all answers on the answer sheet. Only Arabic NUMERICAL answers are needed.**
- **Answer all 25 problems. The total is 120 points. For problems involving more than one answer, full credit will be given only if ALL answers are correct, no partial credit will be given. There is no penalty for a wrong answer.**
- **Diagrams shown may not be drawn to scale.**
- **No calculator or calculating device is allowed.**
- **Answer the problems with pencil, blue or black ball pen.**
- **All papers shall be collected at the end of this test.**

Section A (4 marks each)
A 组 (每题 4 分)

1. What is the value of : $496 \times (\frac{384}{768})^2$

   求 $496 \times (\frac{384}{768})^2$ 之值。

2. The opposite faces of a dice add to seven. The dice below rotate in the directions indicated, one face at a time. There are three dice, after three moves, what will be the total of the front faces?
   每一个骰子的相对两面的点数和是7。若依照箭头的指示方向旋转骰子，每次旋转一面，经过三次旋转后，三个骰子的正面点数之和是多少？

   First   Second   Third

   Front face
   正面
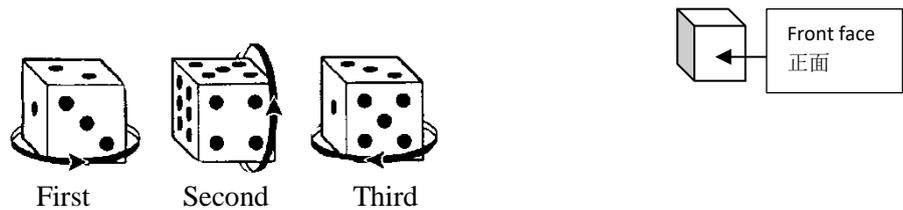
3. Use permutation of 1, 2, 3, 4, 5 to form a five-distinct-digit number, how many of them are multiple of 25?

   将 1，2，3，4，5重新排列组成一个不重复数字的五位数，有多少个是 25 的倍数？

4. In the diagram, ABCD is a parallelogram, AD = AE, BAE is a straight line, ∠ ABC = 88° , ∠ EAF = 162° , and ∠ AFC = 48°. Calculate ∠ AED and ∠ BCF.

   如图所示，ABCD为一平行四边形，AD = AE, BAE 是一直线，∠ ABC = 88° , ∠ EAF = 162° , ∠ AFC = 48°。求∠ AED 和 ∠ BCF.
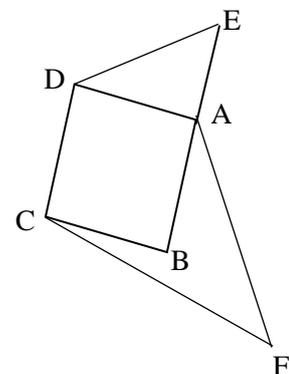
5. Amateur members of a shooting team shot 2018 times in a contest. In the first 2012 times shooting, they shot 14084 rings (The rings are integers.). If they wanted to have the results of 14141 rings, how many rings must they get at least for the 2013rd shooting?

   某业余射击运动队的队员们在一次比赛共射击 2018 次，前 2012 次射击共中 14084 环（环数均为整数），如果他们全队想取得不低于 14141 环的成绩，那么第 2013 次射击不能少于多少环？
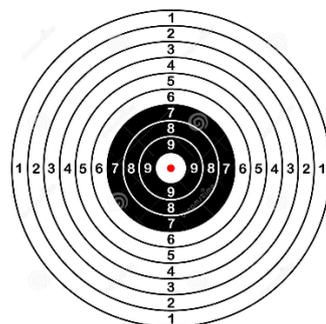
6. A group of students lined up in a straight line from left to right as they counted off numbers 1, 2, 1, 2, 1, 2, … as shown in the first picture and then they would do the same again with numbers 1, 2, and 3 as shown in the second picture. If there were 5 people counted off the number "2" on both occasions, what is the most number of students this group could have?

   一组学生排成一排，从左到右开始依序报数，1，2，1，2，1，2，... 如图（1）所示般报数。之后，他们从左到右如图（2）般，1，2，3，1，2，3，... 报数。如果有5位学生报了两次"2"，这组学生最多有多少人？

   图（1）

   图（2）
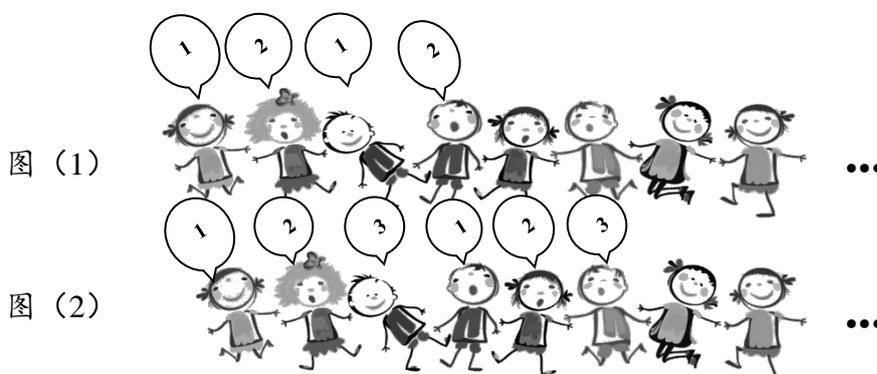
7. There are 48 green, blue and yellow cards in a box. 16 of them are blue. If a card is chosen at random from the box, the probability of choosing a yellow card is $\frac{1}{4}$. Find the number of green cards in the box.

   盒子里有绿色、蓝色和黄色的卡片共 48 张，其中蓝色卡片有 16 张。若任意从盒中抽选一张卡片，这张卡片是黄色的机会是 $\frac{1}{4}$；那么，盒中原有多少张绿卡片？

8. Compute：$\dfrac{20182017^2}{20182016^2 + 20182018^2 - 2} = ?$

   计算 $\dfrac{20182017^2}{20182016^2 + 20182018^2 - 2} = ?$

9. In a 25 × 25 grid paper, numbers 1∼ 9 were filled into each grid (numbers could be repeated), the sum of all the 9 numbers in any cross shaped like ⊹ was being calculated, At least how many sum are having the same answer ?

在 25×25 的方格纸中，将1～9这九个数字填入每个小方格中（数字可以重复），并对所有形如 ⊹ 的"十"字图形中的 9 个数字求和。对于小方格中的数字的任意一种填法，其中和数相等的"十"字图形至少有多少个？

10. Dr Yan drives along a long stretch of road on which a regular service is in operation. He is driving at a constant speed. He observes that every 3 minutes he meets a train and every 6 minutes a train overtakes him. How often does a train leave the terminal station at one end of the route?
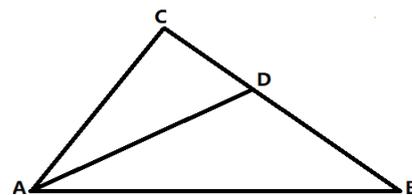
袁医生沿着直路均速行驶拟做例常的服务工作。从起站至终站，他发现每3分钟与一列车迎面相遇；每6分钟有一列车从后超越他。那么，终站每隔多久时间开出一班列车？

Section B (5 marks each)
B 组 (每题 5 分)

11. As shown below, $\angle C = 90^0$, $\angle CAD = \angle BAD$, $CD = 1.5$ cm, $BD = 2.5$ cm. What is the length of AC ?

下图三角形ABC中，已知∠C = 90°，∠CAD = ∠BAD，CD 长 1.5 cm，BD 长 2.5 cm。求AC的长。

12. Turn the pure circulating decimals $0.\dot{a}b\dot{c}d$ into the simplest proper fraction and the sum of its numerator and denominator is 2018. What is the value of $a + b + c + d$?

纯循环小数 $0.\dot{a}b\dot{c}d$ 化成一个最简真分数后,分子与分母的和是 2018，那么 $a + b + c + d$ 的值是多少?

13. In the diagram, O is the center of the square, OA = OC = 2, AB = CD = 4,
CD is perpendicular to OC, which is perpendicular to OA, which in turn is perpendicular
to AB. The square has an area of 64 cm².

(a) Compute the area of trapezoid ABCO.
(b) Compute the area of quadrilateral BCDE.

如图，O是正方形的中心点，OA = OC = 2，AB = CD = 4，
CD垂直OC，OC 垂直OA，OA垂直AB。正方形的面积是
64 cm²。
（1）求梯形ABCO的面积；
（2）求四边形BCDE的面积。
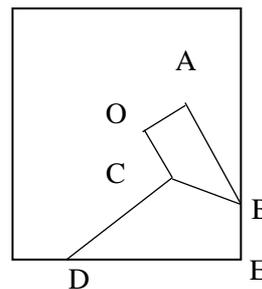
14. John wants to make a 900 ml of 15% alcohol solution by mixing a quantity of a 20% alcohol solution
with a 5% alcohol solution. What are the quantities of each of the two solutions (20% and 5%) he has to
use?

约翰想配制成浓度 15%的酒精溶液 900 ml，他需要浓度 20%与 5%的酒精溶液各多少 ml？

15. A 6-digit number satisfy the property that each digit, from left to right starting from the third digit,
is the product of the two digits before it. How many 6-digit numbers satisfy this property?

一个六位数，每个数字都按规律排列：从左至右的第 3 个数字开始，每个数字是它的前面两个
数字的乘积。符合这个条件的六位数有几个？

16. You are going up a moving upwards escalator, with a speed of 1 step per second. After 50 steps, you
are at the end. You are then turn around and run downwards with a speed of 5 steps per second. After
125 steps, you are back at the beginning of the escalator. How many steps do you need if the escalator
stands still?

你沿着上行的手扶梯向上行走，每秒走1阶，行走了50阶便到达顶部。接着，你转身沿原来的手
扶梯，以每秒5阶的速度向下行走，行走了125阶到达手扶梯底。问：手扶梯静止时露出的阶梯
数是多少阶？

17. A and B are positive integers, and *3A+21B =117*. Find the number of distinct positive integers
solution.

已知A 和 B 都是正整数，且 *3A+21B =117*。那么A, B 有多少组不同的正整数解。

18. If the 6-digit $\overline{20AB18}$ can be exactly divided by 11 and 19 at the same time, what are the values of A
and B respectively?

若六位数 $\overline{20AB18}$ 能同时被 11 和 19 整除，那么 A、B 的值分别是多少？

19. There are some fish in a pond. Whether it is to net seven fish in every catches, or to net eight fish in every catches, there will always five fish left. From the owner's estimation, there are 100 to 150 fish in the pond. Exactly how many fish are there in the pond?

一水池内有些鱼，无论是把鱼每7条一把地抓起来或是每8条地抓起来，水池内总会剩下五条鱼。在水池的主人估算之下，他知道池中的鱼数量在100条至150条之间。根据以上条件，请问水池内有多少条鱼？

20. The shaded octagon shown here is formed from two identical overlapping squares. Each square has sides of 8 cm and the area of each triangle is 2 cm². What is the area of the octagon?

阴影部分的八边形，是由两个相同的正方形所组成的。每个正方形的边长都是8 cm，且每个三角形的面积都是 2 cm²。这个八边形的面积是多少？
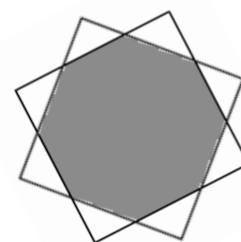
Section C (6 marks each)
C 组 (每题 6 分)

21. What is the sum of the digits of the product $64^{2018} \times 125^{4034}$?

求 $64^{2018} \times 125^{4034}$ 的乘积的各个位数（数码）的和是多少？

22. You have lost in a jungle, and come across five elves. They all look the same, but you have a feeling that some of them are good elves, and always tell the truth, while some of them are bad elves, and always lie. Unfortunately, there is no way to tell them apart just by looking at them – they all look cute and helpful!

   • The first elf says: *I am a good elf!*
   • The second elf says: *At least three of us are good elves!*
   • The third elf says: *Careful! The first elf is a bad elf!*
   • The fourth elf says: *At least three of us are bad elves!*
   • The fifth elf says: *We are all bad elves!*

So, how many elves are good, and how many elves are bad?

你在森林里迷路了，遇到 5 位精灵。他们的长相一模一样，但是你的第六感告诉你，当中有些是好的精灵，有些是坏的精灵；好的精灵永远诚实，而坏的精灵却永远说谎。但是你不知如何分辨好的精灵与坏的精灵，因为他们看起来都是那么的可爱与乐于助人！

第一位精灵 ： 我是好精灵！

第二位精灵 ： 我们当中至少有三位好精灵！

第三位精灵 ： 小心，第一位精灵是坏精灵！

第四位精灵 ： 我们当中至少有三位坏精灵！

第五位精灵 ： 我们都是坏精灵！

到底共有多少位好精灵？多少位坏精灵？

23. Some graduation test papers were checked. On the first day, the ratio of the checked papers and the unchecked papers was 1:4. On the 2nd day, 2018 papers were checked and it is now exactly $\frac{1}{3}$ of the test papers have been checked. What is the total number of these test papers ?

老师对一批毕业考试试卷进行复查评审，第一天已经复查评审的试卷份数与剩下试卷份数比是 1:4，第二天老师们又复查评审了 2018 份试卷，正好完成了这批复查评审试卷份数的 $\frac{1}{3}$ ，那么这批复查评审的毕业考试试卷一共有多少份？

24. Find the tens digit in the value of

$$\frac{4\times 5 \times 6 \times 7 \times ... \times 2017 \times 2018}{\underbrace{5\times 5 \times 5 \times ... \times 5 \times 5}_{500\ "5"}} = ?$$

求下式商数的十数位的数字：

$$\frac{4\times 5 \times 6 \times 7 \times ... \times 2017 \times 2018}{\underbrace{5\times 5 \times 5 \times ... \times 5 \times 5}_{500\ 个\ "5"}} = ?$$

25. How many ways can you arrange four different Chinese books and two different English books on the shelf such that one of the English books must be the first or the last book ?

要将 4 本不同的华文书和 2 本不同的英文书，排放在书架上，如果英文书要排放在第一或者最后，多少种不同的排法？

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